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$$\begin{aligned}
KQ &= \frac{+15 - \sqrt{17} + \sqrt{34 + 2\sqrt{17}} - \beta}{16}, \\
LQ &= \frac{+15 - \sqrt{17} + \sqrt{34 + 2\sqrt{17}} + \beta}{16}, \\
LN &= \sqrt{AL \cdot LQ} = \frac{1}{8} \sqrt{+34 + 2\sqrt{17} + 2\sqrt{34 + 2\sqrt{17}} - 2\beta'}, \\
KM &= \sqrt{AK \cdot KQ} = \frac{1}{8} \sqrt{+34 + 2\sqrt{17} + 2\sqrt{34 + 2\sqrt{17}} + 2\beta'}, \\
[\beta' &= \sqrt{2(\sqrt{17} - 3)(2\sqrt{17} - \sqrt{34 + 2\sqrt{17}})}; \quad \text{and} \quad \beta \times (-1 - \sqrt{17} + \sqrt{34 + 2\sqrt{17}}) = 4 \cdot \beta'], \\
LN \times KM &= \frac{1}{64} \sqrt{2(\sqrt{17} + 3)(2\sqrt{17} + \sqrt{34 - 2\sqrt{17}})} = \frac{4\alpha}{64}, \\
NM^2 &= LK^2 + (KM - LN)^2 \\
&= 4 \cdot EH^2 + KM^2 + LN^2 - \frac{8\alpha}{64} \\
&= \frac{+136 - 8\sqrt{17} + 8\sqrt{34 - 2\sqrt{17}} - 8\alpha}{64}, \\
[\alpha' &= \sqrt{2(\sqrt{17} + 3)(2\sqrt{17} - \sqrt{34 - 2\sqrt{17}})}; \quad \text{and} \quad 4\alpha = \alpha' \times (-1 + \sqrt{17} + \sqrt{34 - 2\sqrt{17}})], \\
\therefore NM^2 &= \frac{+136 - 8\sqrt{17} + 8\sqrt{34 - 2\sqrt{17}} - 2(-1 + \sqrt{17} + \sqrt{34 - 2\sqrt{17}})\alpha'}{64}.
\end{aligned}$$

And, in the circle with radius equal to unity, NM represents the value $2 \sin (2\pi/17) \times 1$; and therefore

$$\begin{aligned}
4 \cos^2 \frac{2\pi}{17} &= 4 - NM^2 \\
&= \frac{+120 + 8\sqrt{17} - 8\sqrt{34 - 2\sqrt{17}} + 2(-1 + \sqrt{17} + \sqrt{34 - 2\sqrt{17}})\alpha'}{64} \\
&= \left[\frac{-1 + \sqrt{17} + \sqrt{34 - 2\sqrt{17}} + \alpha'}{8} \right]^2
\end{aligned}$$

and

$$2 \cos \frac{2\pi}{17} = \frac{-1 + \sqrt{17} + \sqrt{34 - 2\sqrt{17}} + \sqrt{2(\sqrt{17} + 3)(2\sqrt{17} - \sqrt{34 - 2\sqrt{17}})}}{8}.$$

And this value of $2 \cos (2\pi/17)$ will be found to agree with that given in Klein's *Famous Problems in Elementary Geometry* (Beman & Smith), though our α' is there written in a different form.

II. HISTORICAL NOTE BY R. C. ARCHIBALD, Brown University.

This method of construction is due to H. W. Richmond, *Quarterly Journal of Mathematics*, Volume 26, 1893, pp. 206-207; and *Mathematische Annalen*, Volume 67, 1909, pp. 460-461. It is reproduced on page 34 of H. P. Hudson's *Ruler and Compasses*, London, 1916.

Various constructions of the regular polygon of seventeen sides were reviewed by R. Goldenring in his *Die elementargeometrischen Konstruktionen des regelmässigen Siebzehnecks* (Leipzig, 1915), but many omissions in this professedly complete survey were noted by the writer in the *Bulletin of the American Mathematical Society*, vol. 22, 239-246. The first solution in an English publication, given by Lowry in 1819,¹ was reproduced in this *Monthly*² in 1899 and 1914. Other solutions and historical notes are set forth in the articles printed above, pages 322-326.

2767 [1919, 171]. Proposed by W. W. JOHNSON, U. S. Naval Academy.

Let the complex quantities p , q , and r satisfy the relation $p^2 + q^2 + r^2 = 0$; prove that the corresponding vectors OP , OQ , and OR are such that if any two of them are taken as conjugate semi-diameters of an ellipse, the third lies on the minor axis, and its length is the distance from the center to either focus.

SOLUTION BY A. PELLETIER, Montreal, Can.

Let $(x^2/a^2) + (y^2/b^2) = 1$, be the equation of the ellipse having OP and OQ for conjugate semi-diameters ($2a$ and $2b$ being the axes, and $a \geq b$). If α , α' , α'' are the respective arguments of

¹ *The Mathematical Repository*, new series, vol. 4, p. 160; Lowry's proof occupies pages 160-168.

² Volume 6, p. 239 and volume 21, p. 252.

OP , OQ , and OR , we have

$$OP^2(\cos 2\alpha + i \sin 2\alpha) + OQ^2(\cos 2\alpha' + i \sin 2\alpha') = -OR^2(\cos 2\alpha'' + i \sin 2\alpha''),$$

from datum; hence,

$$OP^2 \cos 2\alpha + OQ^2 \cos 2\alpha' = -OR^2 \cos 2\alpha'' \quad (1)$$

and

$$OP^2 \sin 2\alpha + OQ^2 \sin 2\alpha' = -OR^2 \sin 2\alpha''. \quad (2)$$

Now P and Q being points on the ellipse, we have from known properties,

$$OP^2 \cos^2 \alpha + OQ^2 \cos^2 \alpha' = a^2, \quad OP^2 \sin^2 \alpha + OQ^2 \sin^2 \alpha' = b^2;$$

hence, $OP^2 \cos 2\alpha + OQ^2 \cos 2\alpha' = a^2 - b^2$, and (1) becomes

$$-OR^2 \cos 2\alpha'' = a^2 - b^2. \quad (3)$$

Also, from known properties concerning the ends of conjugate diameters,

$$OP^2 \sin 2\alpha = -OQ^2 \sin 2\alpha';$$

hence, (2) becomes

$$-OR^2 \sin 2\alpha'' = 0. \quad (4)$$

It follows from (3) and (4), that $2\alpha'' = 180^\circ$ or 540° , and $OR^2 = a^2 - b^2$, that is, $OR = \sqrt{a^2 - b^2}$, the distance from the center to focus, and $\alpha'' = 90^\circ$ or 270° , which shows that OR lies on the minor axis.

Also solved by H. HALPERIN, A. M. HARDING, and H. L. OLSON.

2780 [1919, 311]. Proposed by ELMER LATSHAW, West Philadelphia, Pa.

A quadrilateral whose sides are $a, 2a, 3a, 4a$ is inscribed in a circle. Find the radius of the circle.

I. SOLUTION BY H. S. UHLER, Yale University.

The interest in this problem may be enhanced by giving a perfectly general solution. Let the sides of any convex inscriptible quadrilateral be denoted by a_1, a_2, a_3, a_4 . A diagonal c may be drawn dividing the quadrilateral into two non-overlapping triangles the sides of which are a_1, a_2, c and a_3, a_4, c , respectively. If the angle between a_1 and a_2 be symbolized by C , the angle between a_3 and a_4 must be $180^\circ - C$. Accordingly

$$c^2 = a_1^2 + a_2^2 - 2a_1a_2 \cos C,$$

$$c^2 = a_3^2 + a_4^2 + 2a_3a_4 \cos C.$$

Eliminating $2\cos C$ we find

$$c^2 = \frac{(a_1a_3 + a_2a_4)(a_2a_3 + a_4a_1)}{a_1a_2 + a_3a_4}. \quad (1)$$

The area of a plane triangle having the sides a_1, a_2, c is given by either member of the following equation

$$\frac{a_1a_2c}{4R} = \sqrt{s(s-a_1)(s-a_2)(s-c)}, \quad (2)$$

where $2s = a_1 + a_2 + c$, and R denotes the radius of the circumscribed circle.

Substituting the trinomial value of s in equation (2) we obtain

$$\frac{a_1a_2c}{R} = \sqrt{[(a_1 + a_2)^2 - c^2][c^2 - (a_1 - a_2)^2]}. \quad (3)$$

Replacing c in equation (3) by expression (1) we eventually find that

$$R = \frac{\sqrt{(a_1a_2 + a_3a_4)(a_1a_3 + a_4a_2)(a_2a_3 + a_4a_1)}}{\sqrt{(a_2 + a_3 + a_4 - a_1)(a_3 + a_4 + a_1 - a_2)(a_4 + a_1 + a_2 - a_3)(a_1 + a_2 + a_3 - a_4)}}, \quad (4)$$

or

$$R = \frac{1}{4K} \sqrt{(a_1a_2 + a_3a_4)(a_1a_3 + a_4a_2)(a_2a_3 + a_4a_1)}. \quad (5)$$

where if $2S = a_1 + a_2 + a_3 + a_4$, $K = \sqrt{(S - a_1)(S - a_2)(S - a_3)(S - a_4)} = \text{area of quadrilateral.}$